5. **Write proofs of each of the following claims about functions of a positive integer n.**

   a. $n^2 - n \in \Omega(n^2)$

   - State the definition:
     $f \in \Omega(g)$:
     There exist constants $c > 0$ and $N_0 \geq 0$ such that
     $$f(n) \geq c \cdot g(n)$$
     for all $n \geq N_0$.

   - $n^2 - n \geq c \cdot (n^2)$
     $n^2 - n \geq c \cdot (n^2)$
     $n^2(1 - \frac{1}{n}) \geq c \cdot n^2$
     $1 - \frac{1}{n} \geq c$

     Since $n > 0$ (question states that n is a positive integer),
     $1 - \frac{1}{n} < 1$
     then
     $c \leq 1 - \frac{1}{n} < 1$
     and
     $c < 1$

     Now we know c is a constant such that $0 < c < 1$.

   - Let's pick $c = \frac{1}{2}$,
     then
     $1 - \frac{1}{n} \geq \frac{1}{2}$
     and
     $2n - 2 \geq n$
     $n \geq 2$

   b. $n^3 \in o(n^4)$

   - State the definition:
     $f \in o(g)$:
     For every constant $c > 0$ there exists a constant $N_0 \geq 0$ such that
     $$f(n) < c \cdot g(n)$$
     for all $n \geq N_0$.

   - $n^3 < c \cdot n^4$
     $n^3 < c \cdot n \cdot n^3$

$cn > 1$

$n > \frac{1}{c}$ for all $c > 0$

6. **Refer back to question 8 in Tutorial 4, where we discovered the following:**
   - $T_{body} \leq 4i + 4$
   - $T_{test} = 1$
   - $Loop\ will\ execute\ n - 1\ times\ in\ the\ worst - case.$

For the loop body:

$T_{body} \leq 4i + 4$

Since $1 \leq i \leq n - 1$

$4i + 4 \leq 4(n-1) + 4$

$= 4n$

Loop cost:

$(n-1)(4n) + (n - 1 + 1)(1)$

$4n^2 - 4n + n$

$4n^2 - 3n$

Since $n \geq 1$

$4n^2 - 3n \leq 4n^2$

$4n^2 \in O(n^2)$

$4n^2 \leq c \cdot n^2$ if $c \geq 4$

For splitting the sum, refer to page 1152 of CLRS textbook.